Mathematical Model

Step 1. To establish the relationship between the temperature and the concentration of pro-insulin

Firstly, our prior assumption states that every cubic millimeter of the bacteria solution contains constant amount of engineered bacteria, and every engineered bacterium contains constant amount of pro-insulin. In order to help release the pro-insulin in the engineered bacteria, we choose lysosome as an active protein that performs different activities under different temperatures.

As a result, we conduct an experiment to deduce the effect of temperatures on lysosome using the same time interval, which further provide the curve for the lysosome's efficiency on our bacteria. The index used to show the efficiency is the total amount of protein (C_{p_all}) in supernatant after hydration for 3 minutes. The results we got are shown in Fig.1.

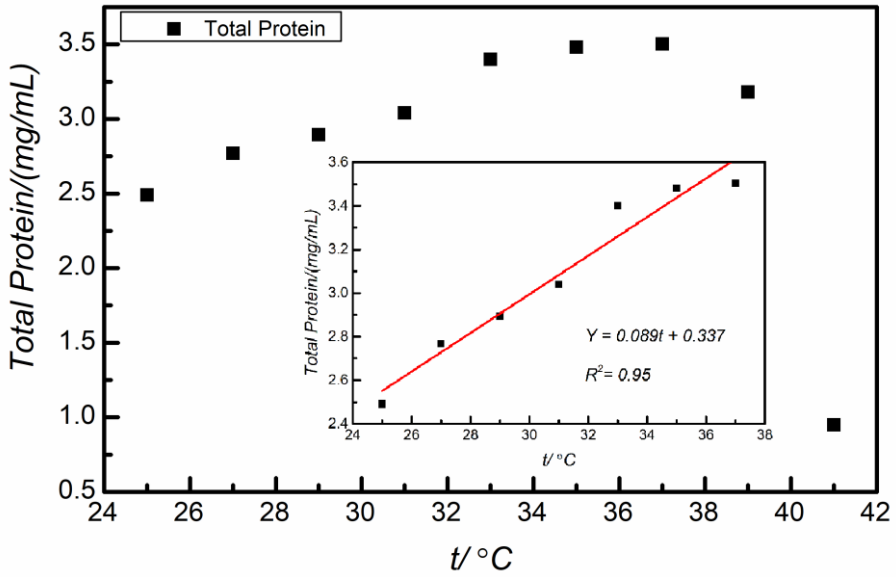

Fig.1 The relationship between temperature and total protein amount

$$C_{p\_all} = 0.089t + 0.337C$$

Step.2 To acquire the change in concentration of glucose caused by pro-insulin

Add glucose with a final concentration of 0.6 mmol/L into the organ-chip, cultivating for 10 minutes under 37°C to ensure the consistency of concentration everywhere in the chip. After the reading of the sensor becomes stable, we measure the concentration of glucose in the liver, providing current signal I_0. After this, we mainly tests the protein obtained by enzymatic hydrolysis at 37 °C in step 1, and tests the effect of reducing glucose concentration by diluting protein to different concentrations in different ratios. The pro-insulin test temperature in the chip is 37 ° C, the test time lasts 20 minutes. The current is shown as I_e. The curved relationship we found was shown in Fig.2.

$$\Delta I = I_0 - I_e$$

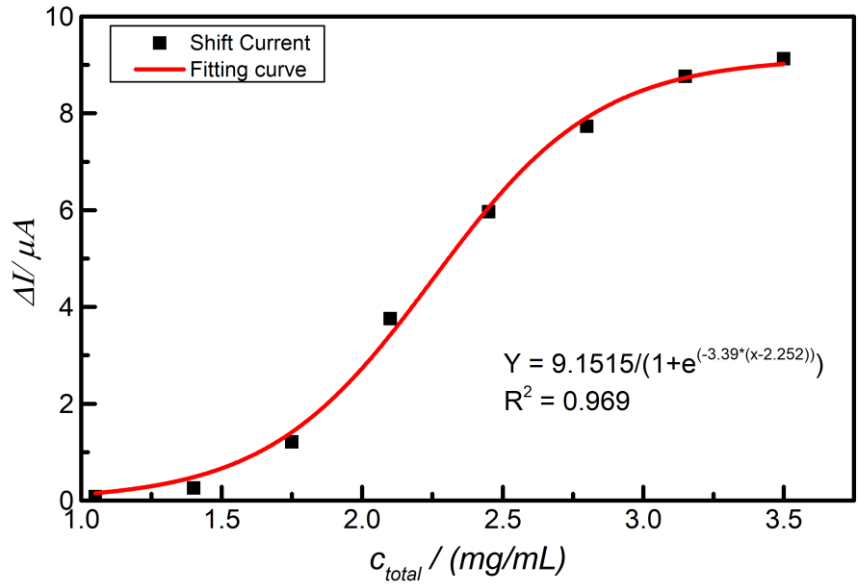

Fig.2 The relationship between concentration of glucose and current shift

$$\Delta I = \frac{9.1515}{1+e^{(-3.39\times(C_{p\_all}-2.252))}}$$

Step 3. To deduce the calibration curve for glucose by our sensor

According to the calibration curve for glucose derived by our electrochemical sensor, the actual glucose concentration change can be calculated by the current signal decrease amount ΔI. The calibration curve for glucose is shown in Fig.3.

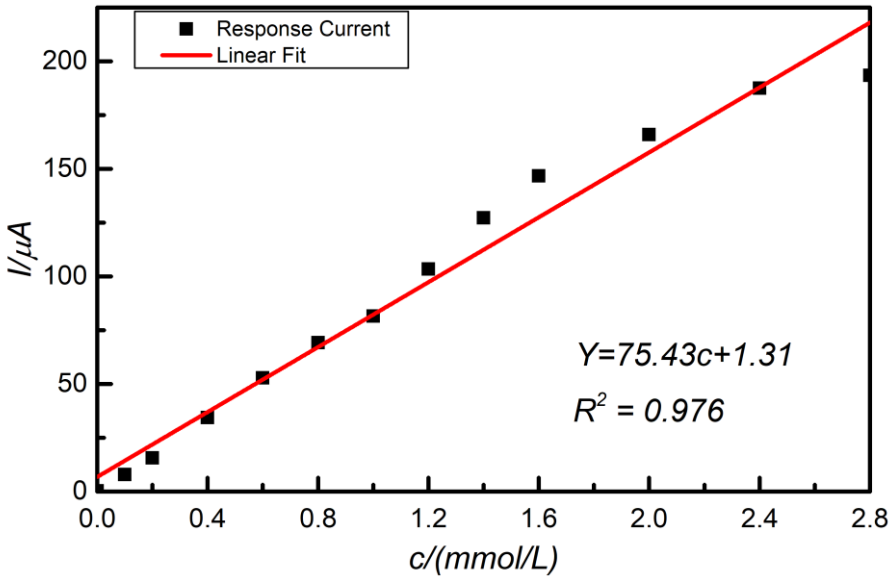

Fig.3 Calibration curve for glucose

$$I = 75.43C + 1.31$$

Step.4 To gain the curve showing the relationship between various concentrations of glucose and current shift under pro-insulin with the same concentration

Eventually, we change the concentration of glucose and detect the current shift under the same temperature, which indicates that the concentration of pro-insulin is not changed. Then we derive the graph showing the relationship between various concentrations of glucose and current shift, telling us the reaction of glucose with different concentrations to the pro-insulin. The relationship is shown in Fig.4.

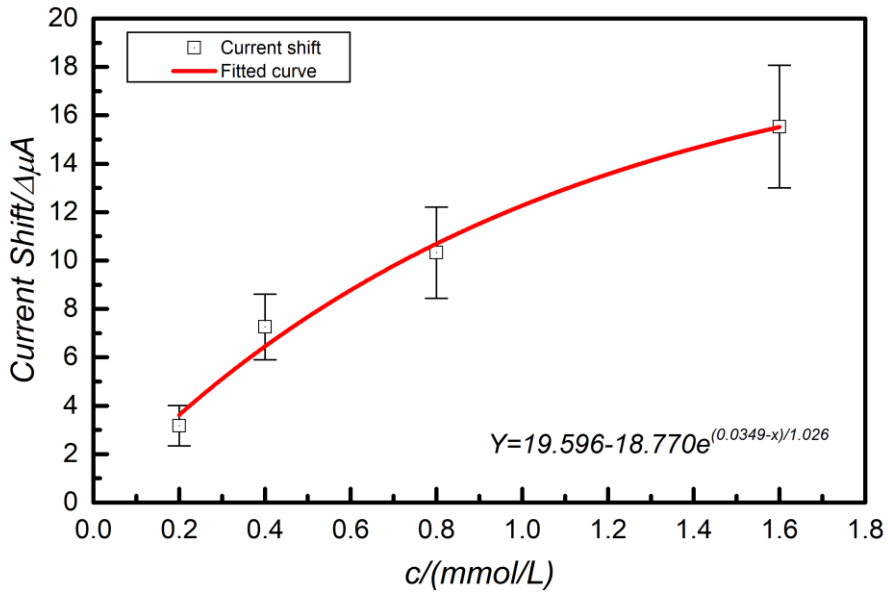

Fig.4 the relationship between various concentrations of glucose and current shift

Now, We mainly have four functions:

Function 1: Indicating the relationship between temperature and concentration of pro-insulin

$$C_{p\_all} = 0.089t + 0.337C$$

Function 2: Indicating the relationship between concentration of pro-insulin and current shift (caused by decrease of glucose concentration)

$$\Delta I = \frac{9.1515}{1 + e^{(-3.39 \times (C_{p\_all} - 2.252))}}$$

Function 3: indicating the relationship between the change in concentration of glucose and the current

$$I = 75.43C + 1.31$$

Function 4: indicating the relationship between the change in concentration of glucose (reacting with pro-insulin of the same concentration) and the current shift

$$\Delta I = 19.596 - 18.770e^{\frac{0.0349 - C}{1.026}}$$

Step 5. To set up the relationship for C_t and t

Firstly, replace the C_{p_all} in function 2 into function 1, which shows the relationship between temperature and concentration of pro-insulin

$$\Delta I = \frac{9.1515}{1+e^{-3.39\times(0.089t+0.337-2.252)}}$$

Differentiate function to see the change in ΔI

$$d\Delta Ip = \frac{2.76e^{(-0.302t+6.49)}}{(1+e^{-0.302t+6.49})^2}dc$$

Then we could see the change in ΔI for every unit increase in temperature.

Secondly, we differentiated function 3 to acquire the change in current for every unit increase in concentration of glucose.

$$dI = 75.43dc$$

Thirdly, we differentiated function 4 to gain the change in ΔI for every unit increase in concentration of glucose (under same concentration of pro-insulin).

$$d\Delta Ig = 18.29e^{\frac{-c+0.0349}{1.026}}dc$$

As there is an opposite effect on current change for adding pro-insulin and adding glucose, we could establish differential equation:

$$d\Delta Ig - d\Delta Ip = dI$$

Finally, this is our graph for $C_{glucose}$ and temperature. The graph is shown in Fig.5

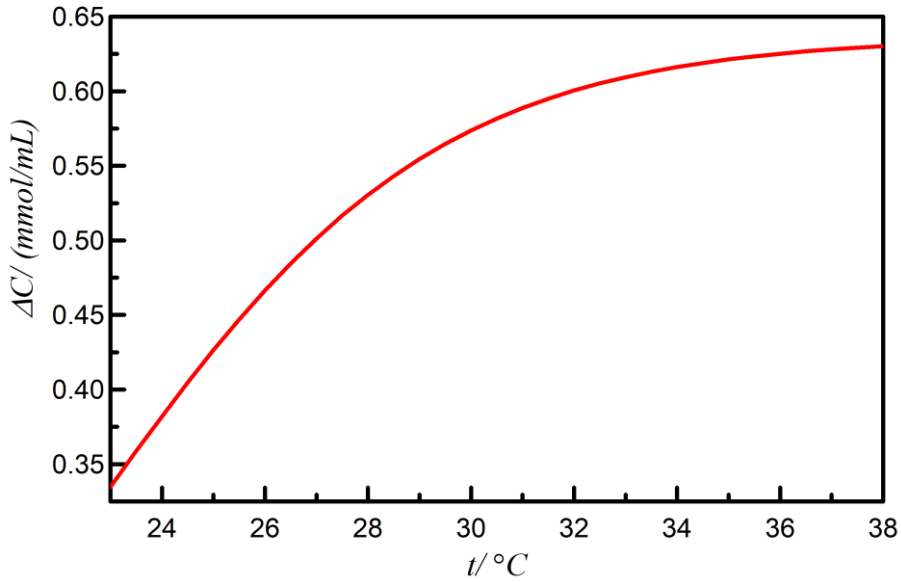

Fig.5 The relationship between glucose and temperature