Differential equations quorum sensing system

The ordinary differential equations are constructed using the Michaelis-Menten model of enzyme kinetics

LuxI

$$\frac{d[mRNALuxI]}{dt} = \alpha_1 + \frac{k_1[LuxR-AHL]}{1+[LuxR-AHL]} - \beta_1[mRNALuxI] \quad (1)$$

$$\frac{d[LuxI]}{dt} = \alpha_2[mRNALuxI] - \beta_2[LuxI] \quad (2)$$

where α_1 is the production constant of LuxI production, and k_1 is the production rate of complex formation LuxR-AHL. The degradation factors are β_1 for degradation of LuxI mRNA, and β_2 the degradation of LuxI protein. The LuxR protein has a

LuxR

$$\frac{d[LuxR]}{dt} = k_2 - \beta_4[LuxR] - \alpha_3[LuxR][AHL] + \beta_3[LuxR - AHL] \quad (3)$$

k_2 production constant and β_4 is the degradation of LuxR. The production constant of regulation by LuxR and AHL is α_3 and β_3 is the dissociation of the complex LuxR-AHL.

Activation:

$$\frac{d[mRNALuxR]}{dt} = \alpha_6 + \frac{\alpha_6[LuxR]}{k7+[LuxR]} - \beta_6[mRNALuxR] \quad (3a)$$

Inhibition:

$$\frac{d[mRNALuxR]}{dt} = \alpha_6 + \frac{k7}{k7+[LuxR]} - \beta_6[mRNALuxR] \quad (3b)$$

The complex formation

Complex formation is governed by

$$\frac{d[LuxR-AHL]}{dt} = \alpha_3[LuxR][AHL] - \beta_3[LuxR - AHL] \quad (4)$$

where α_3 is the production rate of complex formation by LuxR and AHL, β_3 is the dissociation of complex LuxR-AHL.

The change in concentration AHL

The concentration of AHL is governed by

$$\frac{d[AHL]}{dt} = \alpha_4 - \alpha_3[LuxR][AHL] + \beta_3[LuxR - AHL] - k_{so}[AHL] + k_{s1}[LuxI] - \eta[AHL] + k_3 \quad (5)$$

where k_3 is the constant external concentration AHL. The AHL concentration is affected by the degradation rate and AHL synthesis rate. For a single cell the

diffusion rate of AHL over the membrane is $\eta=\sigma A/V_c$ with σ the membrane permeability, and A the surface area, and V_c is the cell volume. Additionally, K_{so} is a degradation parameter and k_{s1} a production parameter.

External AHL concentrations for multiple cells

The external AHL concentration for multiple cells is governed by

$$\frac{d[AHL_{External}]}{dt} = -k_4 - [GFP][AHL_{External}] + k_5 \sum y[AHL] - y[AHL_{External}] \qquad (6)$$

with k_4 and k_5 as production parameters. In the summation y stands for the conditions of the different parameters in all cells.

Subpopulation system

RFP

The RFP concentration is governed by

$$\frac{d[RFP]}{dt} = \frac{\alpha_{10}[\lambda\ cl]}{k_9+[\lambda\ cl]} \frac{k_{10}}{k_{10}+[434\ cl-LVA]} - \beta[RFP]$$

with $\frac{\alpha_{10}[\lambda\ cl]}{k_9+[\lambda\ cl]}$ controlling the activation of lambda cl, and $\frac{k_{10}}{k_{10}+[434\ cl-LVA]}$ the inhibition of 434 cl-LVA, and β the degradation of RFP

λ-cI

The λ-cl concentration is governed by

$$\frac{d[\lambda\ cl]}{dt} = \alpha_{12} \frac{k_{11}}{k_{11}+[LuxR-AHL]} - \beta[\lambda\ cl]$$

with $\alpha_{12} \frac{k_{11}}{k_{11}+[LuxR-AHL]}$ is the inhibition of the luxR-AHL complex on the subpopulation system, β is the degradation of λ-cI.

434 cI-LVA

The concentration of 434-cl-LVA is governed by

$$\frac{d[434\ cl-LVA]}{dt} = \alpha_{12} \frac{k_{11}}{k_{11}+[LuxR-AHL]} - \beta[434\ cl-LVA]$$

with $\alpha_{12} \frac{k_{11}}{k_{11}[LuxR-AHL]}$ controlling the production rate and β is the degradation rate of 434-cI-LVA. We used

$\alpha_{12} \frac{k_{11}}{k_{11}[LuxR-AHL]}$ for the combined system while for the subpopulation we used

$\alpha_{12} \frac{k_9(1+[glucose]+[arabinose]}{k_9)(1+[glucose]+[arabinose])+[AraC](1+[glucose])}$

An extra equation for the function of AraC in the system is

$$\frac{d[AraC]}{dt} = \alpha_{ara} - \beta_{ara}[AraC]$$

For Figure 10 the parameters 1 until 19 correspond to the parameters from the equations, you can find these in table 1:

Table 1: different parameters from Figure 10 corresponding to the parameters from the equations

parameter	parameters
1	α1
2	α2
3	α3
4	α4
5	α5
6	β1
7	β2
8	β3
9	β4
10	β5
11	k1
12	k2
13	ks0
14	ks1
15	k3
16	n
17	k4
18	k5
19	k6