What is the number of ATP molecules that can be produced per second as a function of light irradiance that hits the bacterial membrane?

Once a photon is absorbed by proteorhodopsin (PR), PR must complete its photocycle before it can absorb another photon [1]. At high light irradiance, this leads to saturation. For this we can use the Michaelis-Menten kinetics. V_{max} is the maximum rate of the system and the Michaelis-Menten constant, K_m, is the substrate concentration at which the reaction rate is $\frac{1}{2}V_{max}$.

Walter et al. demonstrated that the system is analogous to a circuit (figure 1), in this circuit representation; the proteorhodopsin (PR) acts like a battery with internal resistance. [2][3]

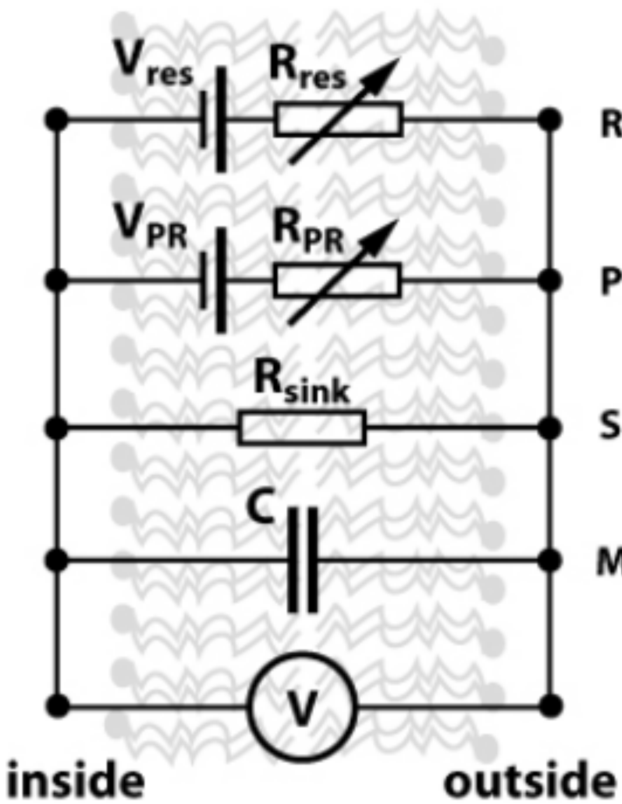

Figure 1, the electric circuit analogy for the membrane.[2]

The current through the system is inversely related to the PR resistor and is dependent on light irradiance.

$$R_{PR} = \left(\frac{V_{max} * I}{K_m + I}\right)^{-1}$$

Walter et al. determined that V_{max} is fixed by the boundary condition that $R_{PR} \approx \frac{R_{sink}}{10}$ at the highest light irradiance, $I = 160 mW/cm^2$. $R_{sink} \approx R_{res} \approx$

$10^{15}\Omega$ and $K_m = 60mW/cm^2$. Where light irradiance of $20mW/cm^2$ is roughly equivalent to PR absorption from solar illumination at sea level. [2]

At the boundary condition:

$$R_{PR} = \frac{R_{sink}}{10} = 10^{14}\Omega = \left(\frac{V_{max} * I}{K_m + I}\right)^{-1}$$

Hence

$$V_{max} = \frac{K_m + I}{R_{PR} * I} = 1.375 * 10^{-14}\Omega^{-1}$$

The rate of reaction, v, has units of Ω^{-1}; through dimensional analysis we can see that:

$$\left[\Omega^{-1} = \frac{Amps}{Volts} = \frac{coulombs}{second * voltage}\right]$$

The voltage across the PR, $V_{PR} = 0.2\ Volts$ [2] and the charge of a proton is $q = 1.6 * 10^{-19}C$.

Therefore we can work out the number of protons pumped by the PR per second as

$$N_{Proton} = \frac{V_{max}I}{(K_m + I)} * \frac{V_{PR}}{q}$$

If an electron pair is composed of 10 protons and there is a net gain of 2.5 ATP molecules per electron pair then the number of ATP molecules produced per second is simply:

$$N_{ATP} = \frac{1}{4}N_{Proton} = \frac{1}{4} * \frac{V_{max}I}{(K_m + I)} * \frac{V_{PR}}{q}$$

The rate of ATP production per second per bacterium as a function of light irradiance has been plotted in figure 2. From the graph, most ATP production rates per second per bacterium are in the range $10^2 - 10^3$, after 5 minutes of illumination each cell would have produced a net gain of about 10^5 ATP molecules, which agrees with experiment [5].

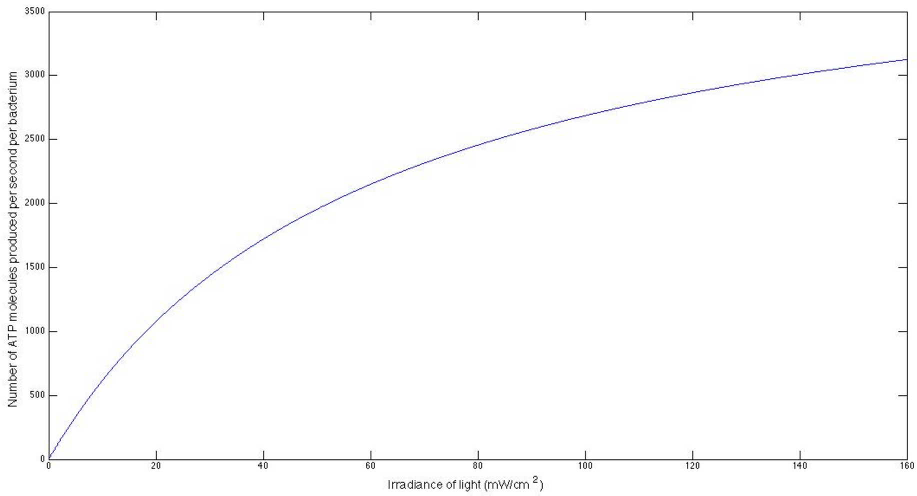

Figure 2: ATP production per second per bacterium as a function of irradiance of light.

Matlab code:

Used a function:

```
function [N_ATP] = atpproduced(I)
%Where I is the irradiance of light in mW/cm^2
V_max = 1.375*10^-14; %Amps/volt
V_PR = 0.2; %Volts, voltage across the PR
K_m = 60; %mW/cm^2, The Michaelis-Menten constant for the reaction
q = 1.6* 10^-19; %C, charge of a proton
N_ATP = 0.25*V_max * I * V_PR / ((K_m + I) * q);

end
```

To produce graph used:

```
I = linspace(0,160,10000);
I = I';
atp = zeros(10000,1);

for iteration = 1:10000
    atp(iteration,1) = atpproduced(I(iteration,1));
end

plot(I,atp)
```

References

[1]Béja, O., Aravind, L., Koonin, E. V., Suzuki, M. T., Hadd, A., Nguyen, L. P., ... & DeLong, E. F. (2000). Bacterial rhodopsin: evidence for a new type of phototrophy in the sea. *Science*, *289*(5486), 1902-1906.

[2] Walter, J. M., Greenfield, D., Bustamante, C., & Liphardt, J. (2007). Light-powering Escherichia coli with proteorhodopsin. *Proceedings of the National Academy of Sciences*, *104*(7), 2408-2412.

[3] Friedrich, T., Geibel, S., Kalmbach, R., Chizhov, I., Ataka, K., Heberle, J., ... & Bamberg, E. (2002). Proteorhodopsin is a light-driven proton pump with variable vectoriality. *Journal of molecular biology*, *321*(5), 821-838.

[4] Cooper, G. M. (2000). Metabolic Energy.

[5] Martinez, A., Bradley, A. S., Waldbauer, J. R., Summons, R. E., & DeLong, E. F. (2007). Proteorhodopsin photosystem gene expression enables photophosphorylation in a heterologous host. *Proceedings of the National Academy of Sciences*, *104*(13), 5590-5595.