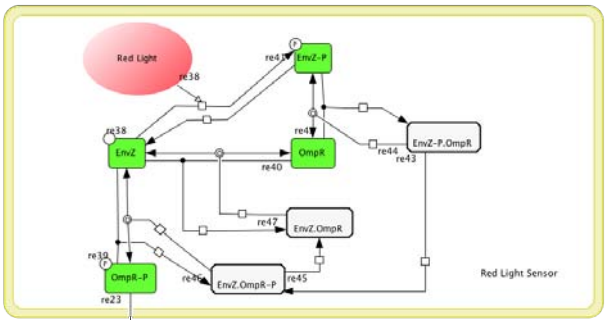
Contents

1. Model

For the Red Light Sensor the approach was to omit the PhyB pathway and guess that the autophosphorelation rate of EnvZ increases linearly with the Intensity of Red Light. If you want to include the PhyB pathway in your model the Paper by Rausenberger [2] should be of great help. The EnvZ/OmpR pathway was modeled according to the Paper by Igoshin [1]. Finally the dye output was an adaption of the model proposed in the paper by Yildirim [4].



If the binding mechanics for the OmpR promoter are of interest, one should consult the paper by Yoshida [3]. They have not been integrated in this model, see the guide for details on how to do that.

2. Equations

$$
\begin{aligned}
EnvZ & \quad \dot{x}_1 = k_{ad}x_2 - k_{ap}x_1 RedLight(t) + k_{d2}x_4 - k_{b2}x_5x_1 - k_{b3} * x_6x_1 + k_{d3}x_7 \\
EnvZ-P & \quad \dot{x}_2 = k_{ap}x_1 RedLight(t) - k_{ad}x_2 + k_{d1}x_3 - k_{b1}x_6x_2 \\
EnvZ-P.OmpR & \quad \dot{x}_3 = -(k_{d1}+k_{pt})x_3 + k_{b1}x_6x_2 \\
EnvZ.OmpR-P & \quad \dot{x}_4 = k_{pt}x_3 - (k_{ph}+k_{d2})x_4 + k_{b2}x_5x_1 \\
OmpR-P & \quad \dot{x}_5 = k_{d2}x_4 - k_{b2}x_5x_1 \\
OmpR & \quad \dot{x}_6 = k_{d1}x_3 + k_{d3}x_7 - k_{b3}x_6x_1 - k_{b1}x_6x_2 \\
EnvZ.OmpR & \quad \dot{x}_7 = k_{ph}x_4 - k_{d3}x_7 + k_{b3}x_6x_1 \\
lacZ_{mRNA} & \quad \dot{x}_8 = \alpha_M \frac{\left(\frac{x_5}{K5}\right)^{n_l}}{1+\left(\frac{x_5}{K5}\right)^{n_l}} - \gamma_M x_8 \\
\beta-Galactosidase & \quad \dot{x}_9 = \alpha_B x_8 - \gamma_B x_9 \\
dye & \quad \dot{x}_{10} = \alpha_A x_9
\end{aligned}
$$

3. Parameters

Parameter	Value	Unit	Name	Source
k_{ap}	0.1	$\frac{1}{s}$	EnvZ autophosphorelation rate	[1]
k_{ad}	0.001	$\frac{1}{s}$	EnvZ dephospholeration rate	[1]
k_{b1}	0.5	$\frac{1}{s}$	binding rate EnvZ-P & OmpR	[1]
k_{d1}	0.5	$\frac{1}{s}$	unbinding rate EnvZ-P.OmpR	[1]
k_{b2}	0.05	$\frac{1}{s}$	binding rate EnvZ & OmpR-P	[1]
k_{d2}	0.5	$\frac{1}{s}$	unbinding rate EnvZ.OmpR-P	[1]
k_{b3}	0.5	$\frac{1}{s}$	binding rate EnvZ & OmpR	[1]
k_{d3}	5	$\frac{1}{s}$	unbinding rate EnvZ.OmpR	[1]
k_{ph}	0.05	$\frac{1}{s}$	dephosphorelation rate EnvZ.OmpR-P	[1]
k_{pt}	1.5	$\frac{1}{s}$	phosphotransfer rate	[1]
$K1$	5	nM	response param. OmpR-P,lacZ	guessed
α_M	$\frac{0.997}{60}$	$\frac{nM}{s}$	max transcription rate lacZ	[4]

Parameter	Value	Unit	Name	Source
α_B	$\frac{1.661e-5}{60}$	$\frac{1}{s}$	max translation rate lacZ	[4]
α_A	$\frac{20}{60}$	$\frac{1}{s}$	enzymatic reaction rate	[4]
γ_M	$\frac{0.411}{60}$	$\frac{1}{s}$	degradation lacZ mRNA	[4]
γ_B	$\frac{8.331e-4}{60}$	$\frac{1}{s}$	degradation β-Galactosidase	[4]

4. Initial Data

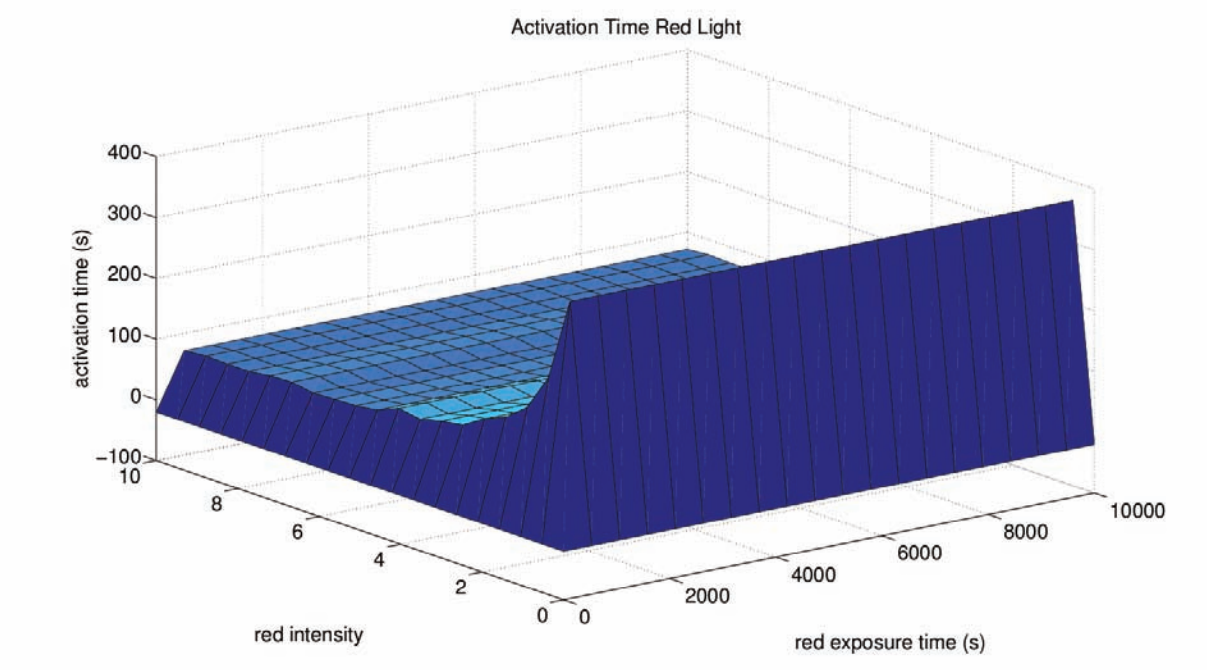
Name	Variable	Initial Value	Comment	Source
$EnvZ$	x_1	$\frac{3500}{0.60221}$	3500 molecules per cell	[1]
$EnvZ - P$	x_2	0		
$EnvZ - P.OmpR$	x_3	0		
$EnvZ.OmpR - P$	x_4	0		
$OmpR - P$	x_5	0		
$OmpR$	x_6	$\frac{100}{0.60221}$	100 molecules per cell	[1]
$EnvZ.OmpR$	x_7	0		
$lacZ_{mRNA}$	x_8	0		
$\beta - Galactosidase$	x_9	0		
dye	x_{10}	0		

5. Simulation

The goal of the simulation was to investigate the dependence of the activation time and deactivation time as well as the output of dye on the irradiation time and the irradiation intensity.

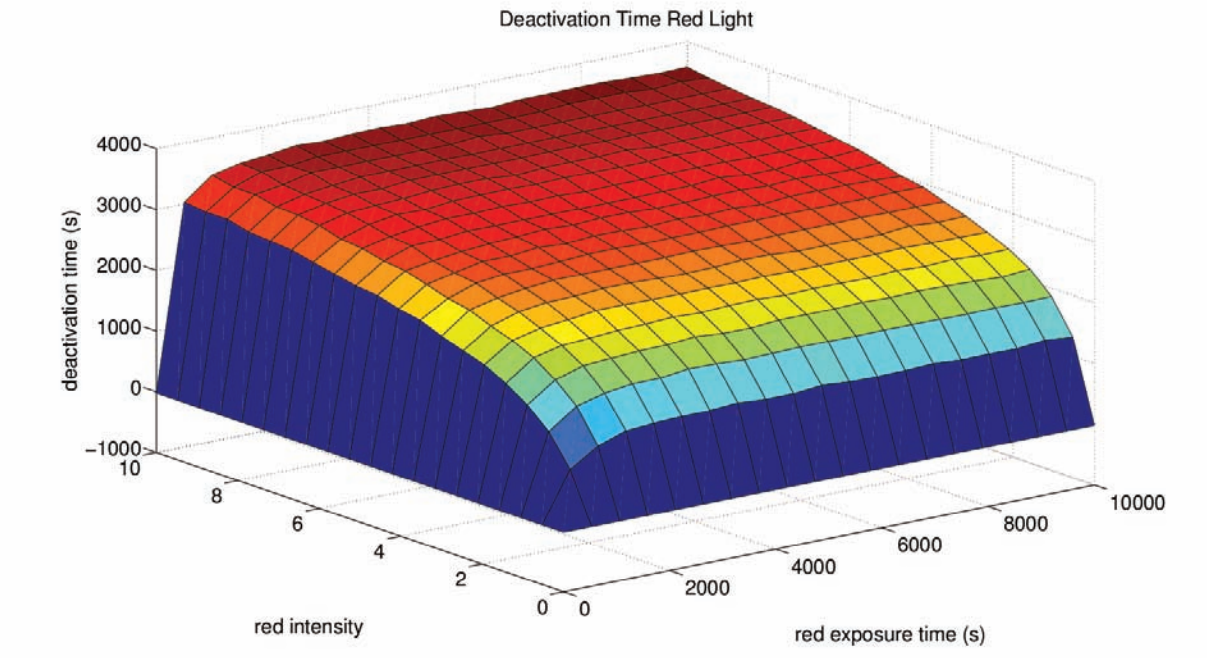
The range of both were adapted such that further increase would lead to no qualitative change.

Irradiation starts immediately at $t = 0$. The activation time is then determined by the time the mRNA concentration exceeds a concentration of $0.5nM$.



We see that the activation time only depends on the intensity of the light, not on the exposure time which should coincide with the real world behavior.

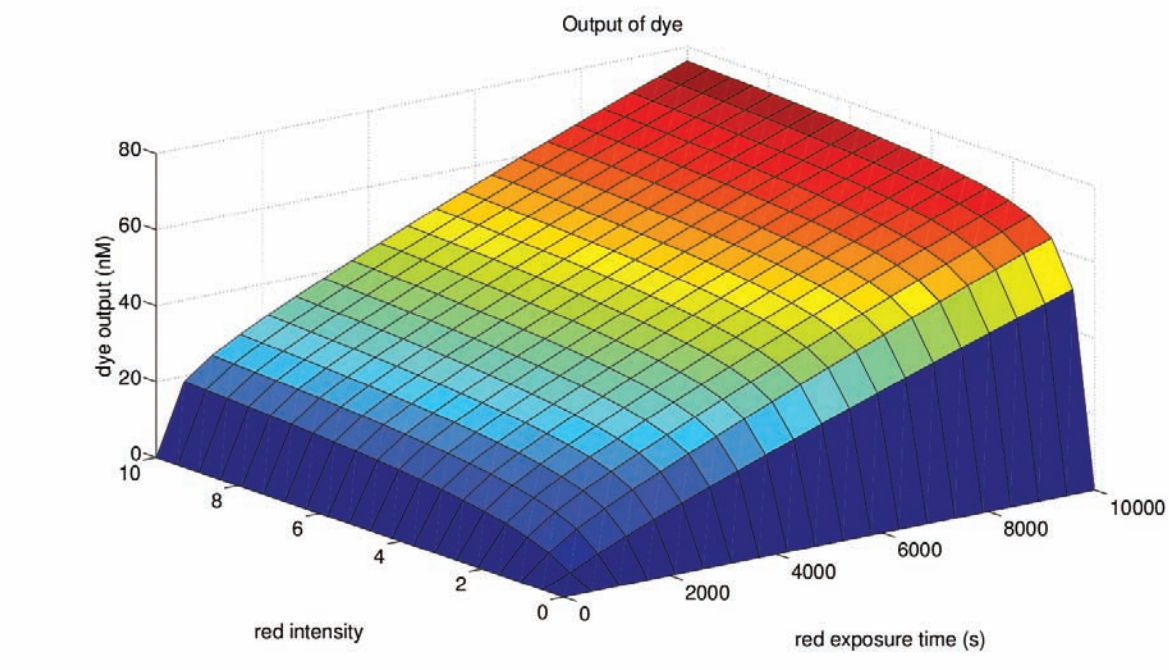
The deactivation time is defined as the timespan that the mRNA concentration needs to again drop below the threshold of $0.5nM$ after expiration of the exposure time.



We see that this deactivation time strongly depends on the intensity and approaches its upper bound only slowly. This is due to the fact that increasing the intensity results in

an increase of the autophosphorylation rate of EnvZ and thus causes a shift the stable state for the phosphorylated OmpR to a higher level. Hence more time is needed to dephosporylate all OmpR and drop the mRNA concentration below the threshold.

The final point of interest is the total output of dye. For this the value of $x_{10}(40000)$ was used. Although this might not be the final output of the system, it should be a rough approximation.



We see that the value depends on both intensity and exposure time, but the upper limit with respect to the intensity is reached quite quickly. The output seems to depend linearly on the exposure time and does not seem to be anywhere near reaching a limit.

6. Conclusion

We can observe that increasing the intensity does increase the deactivation time but does not change the final expression output. For our final system the deactivation time defines the minimum recommended time between two single excitations of focused points. Hence a sufficiently high exposure time at low intensity is desirable.

References

[1] Oleg A Igoshin, Rui Alves, and Michael A Savageau. Hysteretic and graded responses in bacterial two-component signal transduction. *Mol Microbiol*, 68(5):1196–215, Jun 2008.

[2] Julia Rausenberger, Andrea Hussong, Stefan Kircher, Daniel Kirchenbauer, Jens Timmer, Ferenc Nagy, Eberhard Schäfer, and Christian Fleck. An integrative model for phytochrome b mediated photomorphogenesis: from protein dynamics to physiology. *PLoS One*, 5(5):e10721, 2010.

[3] T. Yoshida and L. Qin and L. A. Egger and M. Inouye. Transcription Regulation of ompf and ompc by a Single Transcription Factor Ompr. *Journal of Biological Chemistry*, 281(25):17114–17123, 2006.

[4] N Yildirim, M Santillan, D Horike, and MC Mackey. Dynamics and bistability in a reduced model of the lac operon. *CHAOS*, 14(2):279–292, JUN 2004.